New Block Generalized Averaged Gauss Quadrature Rules

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1 Abstract

Golub and Meurant presented the way of using the symmetric block Lanczos algorithm for calculating the block Gauss quadrature rules for approximating of matrix functions $w^T f(A)u$, where A is a large square matrix, u and v are block-vectors and f is a function. We describe a new block quadrature rules . These rules can be calculated using the symmetric or nonsymmetric block Lanczos algorithms for calculating error estimates for computed quantities, and shows how to achieve more accuracy than standard block Gauss rules for the same computational effort. Our methods are based on block generalizations of the generalized averaged Gauss quadrature rules that were recently proposed by Spalevic.The new representation suggested by Spalevic is a new averaged Gauss quadrature rule that has higher degree of exactness and the same number of nodes as the averaged rule proposed by Laurie. Numerical experiments reported in this paper show the latter averaged rule to yield higher accuracy than Laurie's averaged rule for smooth integrals and, therefore, also can be used to estimate the error in Gauss rules and to approximate integrals. In addition, We describe how to use a symmetric or nonsymmetric adjacency matrix for a network to evaluate functions as applications.

Keywords: Gauss quadrature,Block quadrature rules,Averaged Gauss rule,Generalized averaged Gauss rule.

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