

One Modification of the Chebyshev Measure of the Second Kind and Corresponding Orthogonal Polynomials

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Abstract

The first results to the problem of modification of positive measure with finite support and construction of corresponding orthogonal polynomials were proposed by Christoffel and later by Uvarov (see [2] and [10]). The constructive theory of orthogonal polynomials on the real line was developed in the papers of Gautschi ([6], [7]). Investigations of polynomials orthogonal to non-standard measures and weights have great importance in numerous branches of science and it has been especially important in the development of modified Gaussian quadratures with maximal algebraic degree of exactness ([8]).

Starting from recent results given for example in [4], [3] and [1], in this paper we present our new results on polynomials $\{p_k^{n,s}(x)\}$ orthogonal to one modification of the Chebyshev measure of the second kind given by

$$d\sigma^{n,s}(x) = |\widehat{U}_n(x)|^{2s}(1-x^2)^{1/2+s}dx, \quad n \in N, \quad s > -1/2,$$

where $\widehat{U}_n(x)$ is n -th degree monic Chebyshev polynomial of the second kind. Here we determined the coefficients of the three-term recurrence for observed polynomials $\{p_k^{n,s}(x)\}$ in closed analytic form and derived a differential equality, as well as the differential equation of second order for orthogonal polynomials $\{p_k^{2,s}(x)\}$.

In order to verify all complex formulas we have employed and implemented some symbolic computations in Mathematica, with the intensive use of software package OrthogonalPolynomials ([5], [9]).

Keywords: Orthogonal polynomials, Chebyshev measure, Chebyshev polynomials, recurrence relation, differential equation

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