Ill-posed problems for third order tensors: *tensorize or matricize?*

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Abstract

In this talk, we describe solution methods for linear discrete ill-posed problems defined by third-order tensors and the t-product formalism introduced by Kilmer et al., (2013). We discuss extensions of the standard Arnoldi iteration for matrices to third-order tensors, namely, the t-Arnoldi, global t-Arnoldi, and generalized global t-Arnoldi processes, for which the latter two processes involve flattening. Several algorithms based on these extensions are presented. Solution methods considered are based on computing a few steps of the extended Arnoldi process. Each of the t-Arnoldi-type processes is applied to reduce a large-scale Tikhonov regularization problem for third-order tensors to a problem of small size. Regularization by truncated iterations is considered. This gives rise to extensions of the standard GMRES method, referred to as the tGMRES-type methods. The data is represented by a laterally oriented matrix or a third-order tensor, and the regularization operator is a third-order tensor. The discrepancy principle is used to determine the regularization parameter and the number of steps of the t-Arnoldi-type process. Numerical examples discuss applications to (color) image and video restorations. We compare results for several solution methods and illustrate the potential superiority of solution methods that tensorize over solution methods that matricize linear discrete ill-posed problems for third-order tensors.

Keywords: discrepancy principle, linear discrete ill-posed problem, tensor Arnoldi process, t-product, tensor Tikhonov regularization

References

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