Complex moment-based method with nonlinear transformation for computing partial singular triplets

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Abstract

Given a rectangular matrix $A \in \mathbb{R}^{m \times n}$ $(m \geq n)$, let

$$A = U \Sigma V^{\mathrm{T}} = \sum_{i=1}^{n} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^{\mathrm{T}}$$

be a singular value decomposition of A, where σ_i are singular values and $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are the corresponding left and right singular vectors, respectively, and $U = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n]$, $V = [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n]$ and $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$. To compute partial singular triplets specifically corresponding to the larger part of singular values, there are several projection-type methods such as Golub-Kahan-Lanczos method, Jacobi-Davidson type method and randomized SVD algorithm.

This study considers computing partial singular triplets corresponding to the singular values in some interval,

$$(\sigma_i, \boldsymbol{u}_i, \boldsymbol{v}_i), \quad \sigma_i \in \Omega := [a, b], \tag{1}$$

where $0 \leq a < b$. One of the simplest ideas to compute (1) is to apply some eigensolver for solving the corresponding symmetric eigenvalue problems of $A^{\mathrm{T}}A$ or AA^{T}. However, this simple strategy does not work well in some situation due to the numerical instability.

Based on the concept of the complex moment-based eigensolvers [1, 2], in this study, we propose a novel complex moment-based method to compute partial singular triplets (1). Based on the concept of the complex moment-based parallel eigensolvers, now, we have the following theorem.

Theorem 1. *Let $L, M \in \mathbb{N}_+$ be the input parameters and $V_{\mathrm{in}} \in \mathbb{R}^{n \times L}$ be an input matrix. We define $S \in \mathbb{R}^{n \times LM}$ and $S_k \in \mathbb{R}^{n \times L}$ as follows:*

$$S := [S_0, S_1, \ldots, S_{M-1}], \quad S_k := \frac{1}{2\pi \mathrm{i}} \oint_{\Gamma} z^k (zI - A^{\mathrm{T}}A)^{-1} V_{\mathrm{in}} \mathrm{d}z, \tag{2}$$

where Γ is a positively oriented Jordan curve around $[a^2, b^2]$. Then, the subspaces $\mathcal{R}(AS)$ and $\mathcal{R}(S)$ are equivalent to subspaces with respect to the left and right

singular vectors corresponding to the singular values in a given interval $\Omega = [a, b]$, i.e.,

$$\mathcal{R}(AS) = \mathrm{span}\{\boldsymbol{u}_i | \sigma_i \in \Omega = [a,b]\}, \quad \textit{and} \quad \mathcal{R}(S) = \mathrm{span}\{\boldsymbol{v}_i | \sigma_i \in \Omega = [a,b]\},$$

if and only if $\mathrm{rank}(S) = t$*, where* t *is the number of target singular values.*

This theorem denotes that the singular triplets corresponding to $\sigma_i \in [a, b]$ (1) can be obtained by some projection method with $\mathcal{R}(AS)$ and/or $\mathcal{R}(S)$ constructed by contour integral (2). In practice, the contour integral (2) is approximated by a numerical integration rule such as the N-point trapezoidal rule, as follows:

$$\widehat{S} := [\widehat{S}_0, \widehat{S}_1, \ldots, \widehat{S}_{M-1}], \quad \widehat{S}_k := \sum_{j=1}^{N} \omega_j z_j^k (z_j I - A^{\mathrm{T}} A)^{-1} V_{\mathrm{in}},$$

where $(z_j, \omega_j), j = 1, 2, \ldots, N$, are the quadrature points and the corresponding weights, respectively. Then, the approximate singular triplets are computed by a projection method.

From the analysis of error bounds, we also show that the accuracy of the proposed method can be improved by a nonlinear transformation. Details of the proposed method and numerical results will be reported in the presentation.

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Keywords: Partial singular triplets, Complex moment-based method, nonlinear transformation

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