Accuracy and early termination of Krylov solvers in interior point methods

Filippo Zanetti[1] and Jacek Gondzio[1]

School of Mathematics, University of Edinburgh, Edinburgh, UK
f.zanetti@sms.ed.ac.uk, j.gondzio@ed.ac.uk

Abstract

When an iterative method is applied to solve the linear equation system in Interior Point Methods (IPMs), the attention is usually placed on accelerating their convergence by designing appropriate preconditioners, but the linear solver is applied as a black box solver with a standard termination criterion which asks for a sufficient reduction of the residual in the linear system. Such an approach often leads to an unnecessary "oversolving" of linear equations. In this talk, it is shown how an IPM can preserve the polynomial worst-case complexity when relying on an inner termination criterion that is not based on the residual of the linear system. Moreover, a practical criterion is derived from a deep understanding of IPM needs. The new technique has been adapted to the Conjugate Gradient (CG) and to the Minimum Residual method (MINRES) applied in the IPM context. The new criterion has been tested on a set of quadratic optimization problems including compressed sensing, image processing and instances with partial differential equation constraints, and it has been compared to standard residual tests with variable tolerance. Evidence gathered from these computational experiments shows that the new technique delivers significant improvements in terms of inner (linear) iterations and those translate into significant savings of the IPM solution time.

References

1. Zanetti, F., Gondzio, J.: A new stopping criterion for Krylov solvers applied in Interior Point Methods. arXiv:2106.16090 (2022)