Discretization of the $\star$-Lanczos procedure

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Abstract

The time ordered exponential $U(t)$ is the solution to the ODE

$$\frac{d}{dt}U(t) = A(t)U(t), \quad U(s) = I, \quad t \geq s,$$

where $A(t)$ is a time dependent matrix and s is the starting time. The bilinear form $w^H U(t)v$, with vectors v and w can be approximated by the $\star$-Lanczos procedure. This is a new symbolic algorithm which generalizes the nonHermitian Lanczos iteration to work on bivariate functions. It projects $A(t)\Theta(t-s)$, with Θ the Heaviside step function, onto a Krylov-type subspace which is constructed from bivariate functions using a convolution-like product, the $\star$-product.

We aim to develop a numerical algorithm discretizing the $\star$-Lanczos procedure. This discretized procedure uses a discretization of the $\star$-product and of the related $\star$-inverse.

In this talk we will discuss suitable discretizations and their properties. We mainly focus on an approximation with Legendre polynomials. Such an approximation allows us to transform the $\star$-product between two bivariate functions into the usual matrix-matrix product and the $\star$-inverse into the matrix inverse. The matrices contain the coefficients of the bivariate functions expanded in the Legendre basis. Because of the nature of the problem, the functions of interest are the product of a smooth function and the heaviside step function, i.e., discontinuous, but piecewise smooth. This leads to the Gibbs phenomenon in the discretized functions and corrupts the reconstruction of a function from its coefficients. We show that most of the Legendre coefficients can be computed up to machine precision and they satisfy a decay property, i.e., the coefficient matrices are numerically banded matrices. The properties of these banded Legendre coefficient matrices, and the product of two such matrices, are fundamental to an effective discretization of the $\star$-product and $\star$-inverse and to overcome the Gibbs phenomenon.