A Nyström method for Volterra integral equations based on equally spaced nodes

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Abstract

In this talk, we present a Nyström-type method based on equally spaced points of $[0,1]$, for Volterra integral equation of the type

$$f(s) + \mu \int_0^s k(t,s) f(t) (t-s)^\alpha t^\beta dt = g(s), \quad s \in (0,1),$$

where f is the unknown function, k and g are given functions, $\mu \in \mathbb{R}$, and $\alpha, \beta \geq 0$.

The use of equidistant points is crucial in many engineering and mathematical physics problems which are modeled by Volterra equations, and k and g are available only in a discrete set of equispaced nodes. In all these cases, the classical methods based on piecewise polynomial approximation offer a lower degree of approximation, while the efficient procedures based on the zeros of orthogonal polynomials cannot be used.

Here, we present a Nyström method based on a quadrature formula obtained by means of the sequence $\{B_{m,\ell}(f)\}_m$ of the so called Generalized Bernstein polynomials, where $B_{m,\ell}(f)$ is the ℓ iterated boolean sum of the classical Bernstein polynomial $B_m(f)$, i.e.

$$B_{m,\ell}(f) = f - (f - B_m(f))^\ell, \qquad \ell \in \mathbb{N}, \quad B_{m,1}(f) = B_m(f).$$

$B_{m,\ell}(f)$ requires the samples of f at equally spaced nodes as well as the "original" Bernstein polynomials $B_m(f)$. However, differently from $B_m(f)$, the rate of convergence of $B_{m,\ell}(f)$ improves as the smoothness of the function increases. Indeed, by approximating $f \in C^{2\ell}([0,1])$ by $\{B_{m,\ell}(f)\}_{m,\ell}$, it is $\|f - B_{m,\ell}(f)\|_\infty \sim \mathcal{O}(m^{-\ell})$.

Stability and convergence of the Nyström method are proved in Hölder–Zygmund type spaces and some numerical tests are shown to confirm the theoretical estimates.

Keywords: Nyström Methods, Volterra Integral Equations, Bernstein Polynomials