Some properties of orthogonal polynomials with respect to the Abel weight

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Abstract

While a lot is known about the classical orthogonal polynomials, their counterparts with respect to non-classical weights are not as well explored. Nevertheless, sometimes such weights come handy as well. For example, the famous Abel-Plana summation formula offers a convenient method of summing an infinite series, reducing the sum to an integral with the Abel weight function on the real line,

$$w(x) = \frac{x}{e^{\pi x} - e^{-\pi x}}.$$

Orthogonal polynomials with respect to this weight [1,3] naturally arise when we have to numerically evaluate this integral using the Gauss quadrature rule [2]. These polynomials turn out to possess various symmetry properties. Here we investigate some properties of these polynomials and give some relations between these and some related polynomials.

Keywords: Orthogonal polynomials, Abel weight function, Non-classical weight function

References

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