Regularized Recycling and Subspace Augmentation Methods: Theory and Applications in Adaptive Optics

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Subspace recycling techniques have been used quite successfully for the acceleration of iterative methods for solving large-scale linear systems. These methods often work by augmenting a solution subspace generated iteratively by a known algorithm with a fixed subspace of vectors which are “useful' for solving the problem. Often, this has the effect of inducing a projected version of the original linear system to which the known iterative method is then applied, and this projection can act as a deflation preconditioner, accelerating convergence. In this talk we consider subspace augmentation-type iterative schemes applied to linear ill-posed problems in a continuous Hilbert space setting, based on a recently developed framework describing these methods. We show that under suitable assumptions, a recycling method satisfies the formal definition of a regularization, as long as the underlying scheme is itself a regularization. We then develop an augmented subspace version of the gradient descent method and Tikhonov regularization and demonstrate their effectiveness, both on an academic Gaussian blur model and on problems arising from the adaptive optics community for the resolution of large sky images by ground-based extremely large telescopes. Specifically, we consider wavefront reconstruction from Pyramid sensor data and the atmospheric tomography problem.