Stein-based preconditioners for weak-constraint 4D-var

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Abstract

Algorithms for data assimilation try to predict the most likely state of a dynamical system by combining information from observations and prior models. One of the most successful data assimilation frameworks is the linearized weak-constraint four-dimensional variational assimilation problem (4D-Var), that can be ultimately interpreted as a minimization problem. The linear algebraic problem can be solved by means of a Krylov method, like MINRES or GMRES, that needs to be preconditioned to ensure fast convergence in terms of the number of iterations.

The saddle point formulation of weak-constraint 4D-Var offers the possibility of exploiting modern computer architectures and algorithms due to its underlying block structure. Developing good preconditioners which retain the highly-structured nature of the saddle point system has been an area of recent research interest, especially for applications to numerical weather prediction [3, 1]. In this talk I will present a new preconditioning approach which exploits inherent Kronecker structure within a matrix GMRES implementation [1, 2]. In addition to achieving better computational performance, the latter machinery allows us to derive tighter bounds for the eigenvalue distribution of the preconditioned saddle point linear system. A panel of diverse numerical results displays the effectiveness of the proposed methodology compared to current state-of-the-art approaches.

Keywords: Stein equations, data assimilation, preconditioners

References

1. Freitag, M. A. and Green, D. L. H. A low-rank approach to the solution of weak constraint variational data assimilation problems. Journal of Computational Physics, (2018) 357, 263–281.
2. Stoll, M. and Breiten, T. A low-rank in time approach to PDE-constrained optimization. SIAM Journal of Scientific Computing, (2015) 37, B1–B29.
3. Tabeart, J. M. and Pearson, J. W.: addle point preconditioners for weak-constraint 4d-var. arXiv preprint arXiv:2105.06975