Towards a matrix-free parallelization of the scalable deflation method for the 3D heterogeneous high-frequency Helmholtz equation

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Abstract

The Helmholtz equation arises in many applications such as seismic imaging, sonar, antennas, and medical imaging. It is one of the hardest problems to solve both in terms of accuracy and convergence due to scalability issues of the numerical solvers. Motivated by the observation that for large wavenumbers, the eigenvalues of the preconditioned system with the well-known Complex Shifted Laplacian Preconditioner (CSLP) shift towards zero, deflation techniques were incorporated to accelerate the convergence of preconditioned Krylov subspace methods [1]. Deflation aims at projecting the small eigenvalues to zeros. But the near-zero eigenvalues will reappear for larger wavenumber. In [2, 3], the adapted deflation preconditioning (ADP) methods, using higher-order deflation vectors, combined with CSLP can lead to scalable convergence. Targeting modern large-scale applications, parallelization of the scalable deflation techniques is in progress. As CSLP is the cornerstone, we start with a matrix-free parallel variant of the CSLP preconditioned Krylov subspace methods for the 3D heterogeneous Helmholtz equation. The CSLP is approximately inverted using one parallel multigrid cycle. The matrix-vector multiplications and preconditioning operators are all implemented in a matrix-free way. Numerical experiments of 3D model problems show that the matrix-free parallel solution method has improved parallel performance and weak scalability.

Keywords: Helmholtz equation, Parallel computing, Deflation, Multigrid, Preconditioner, Scalable, Matrix-free

References

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