A Nyström method for 2D linear Fredholm integral equations on curvilinear polygons

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Abstract

The talk deals with the numerical approximation of 2D Fredholm linear integral equations of the type

$$f(x,y) - \mu \int_S k(x,y,s,t) f(s,t) ds dt = g(x,y), \qquad (x,y) \in S,$$

where S is a general bi-dimensional domain whose boundary is a piecewise smooth Jordan curve (curvilinear polygon), g and k are known functions defined on S and S^2, respectively, μ is a fixed real parameter and f is the unknown in S.

For 2D Fredholm integral equations very few methods were proposed. The most considered case is that of rectangular domains (see for istance [1] and the references therein).If a global approximation strategy is chosen, the most simple and powerful approach seems to be Nyström methods based on cubature rules obtained as the tensor product of two univariate rules.

On the contrary no global approach was proposed for the case of curvilinear domains that cannot be transformed in a square. On the other hand even if the transformation is possible, the smoothness of the known functions could be not preserved and this can produce a severe loss in the rate of convergence of the method.

Here we propose a numerical method of Nyström type based on cubature formulas introduced in [2]. This choice allows to consider general curvilinear domains, to use a global approximation approach and avoids the loss of convergence due to transformations.

Keywords: Fredholm integral equations, Nyström method, Curvilinear polygons.

References

1. D. Occorsio, M.G. Russo, *Numerical methods for Fredholm integral equations on the square*, Applied Mathematics and Computation 218 (2011), 2318-2333.
2. G. Santin, A. Sommariva, M. Vianello, *An algebraic cubature formula on curvilinear polygons*, Applied Mathematics and Computation 217 (2011), no. 24, 10003-10015.