A numerical method for hypersingular integrals of highly oscillatory functions on $[0, +\infty)$.

Maria Carmela De Bonis[1] and Valeria Sagaria[1]

Department of Mathematics, Computer Science and Economics,
University of Basilicata, Viale dell'Ateneo Lucano, 85100 Potenza, Italy
{mariacarmela.debonis,valeria.sagaria}@unibas.it

Abstract

We consider the approximation of the following integrals

$$H_p^{\omega,\gamma}(f,t) = \neq\!\!\!\!\!\!\int_0^{+\infty} \frac{f(x)e^{i\omega x}}{(x-t)^{p+1}} u_\gamma(x)\,dx, \tag{1}$$

where $t > 0, p \geq 0$ is an integer, $i^2 = -1$, $\omega \gg 1$, $u_\gamma(x) = x^\gamma e^{-\frac{x}{2}}, \gamma \geq 0$, is a generalized Laguerre weight and the integral is understood in the Cauchy principal value sense if $p = 0$ and in the finite part Hadamard sense if $p > 0$.
Integrals $H_p^{\omega,\gamma}(f,t)$ are of great interest in many scientific areas, such image analysis, optics, electrodynamics and electromagnetism. In particular, they frequently appear in boundary element methods and their efficiency often depends upon the accuracy of the numerical evaluation of (1).
To our knowledge a wide literature is available on the approximation of singular and hypersigular integrals for non oscillatory function. The same is true for integrals of non singular highly oscillatory functions. Moreover, most of the papers present in literature about integrals having both singularity and oscillation are devoted to the case of bounded intervals, and very few references deal with $H_p^{\omega,\gamma}(f,t)$ with $p = 0$.
In order to evaluate (1) we propose a product quadrature rule based on the approximation of function f by a truncated Lagrange polynomial using the Laguerre zeros and the additional point $4m$. Stability and convergence are proved in suitable weighted Sobolev type spaces. Comparison with the results obtained using the methods introduced in $[1, 2]$ are also given. Finally we present an application of the proposed quadrature rule for the construction of a Nyström type method for the numerical solution of the following integral equations

$$f(t) + \neq\!\!\!\!\!\!\int_0^{+\infty} \frac{f(x)e^{i\omega x}}{(x-t)^{p+1}} u_\gamma(x)\,dx = g(t), \quad t > 0,$$

where g is a given function and f is the unknown.

Keywords: Product rule, Hadamard integrals, Cauchy integrals, Nyström methods

References

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