Error term in Gauss quadrature with Legendre weight function for analytic functions

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Abstract

We studied the error bound of Gauss-Legendre quadrature for analytic functions. It is well known when f is an analytic function then remainder term can be represented as contour integral with a complex kernel.

$$R_n(f) = I(f;\omega) - \sum_{i=1}^{n} \omega_i f(\xi_i) = \frac{1}{2\pi\, i} \oint_{\Gamma} K_n(z;\omega) f(z)\, dz,$$

where

$$I(f;\omega) = \int_{-1}^{1} \omega(t)\, f(t)\, dt, \quad \omega_i = \int_{-1}^{1} \omega(t)\, l_i(t)\, dt.$$

Then the error bound is reduced to find the maximum of the kernel function. $K_n(z) = K_n(z;\omega)$ can be expressed in the form

$$K_n(z;\omega) = \frac{\varrho_n(z;\omega)}{\Omega_n(z)}, \quad \varrho_n(z;\omega) = \int_{-1}^{1} \omega(t)\, \frac{\Omega_n(t)}{z-t}\, dt, \quad z \in C \setminus [-1,1].$$

We derived explicit expression of the kernel function $K_n(z;\omega)$ and analysed behaviour of this expression in order to determine points where maximum is attained.

Keywords: Gaussian quadratures, Legendre polynomials, Error bound

References

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