Finite difference approximation of a initial-boundary value problem with Dirac delta function and time-dependent coefficients

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Abstract

One interesting class of parabolic problems model processes in heat-conduction media with concentrated capacity in which the heat capacity coefficient contains a Dirac delta function. In this paper we study the convergence of a finite difference scheme that approximates the initial-boundary problem for the heat equation with concentrated capacity and time-dependent coefficients. We assume that the generalized solution of the problem belongs to the Sobolev space $W_2^{s,s/2}, 5/2 < s \leq 3$. Order convergence rate estimate in the discrete $W_2^{1,1/2}$ norm is obtained. The result is based on some nonstandard a priori estimates involving fractional order discrete Sobolev norms.

Keywords: Partial differential equations, Delta function, Sobolev space, Convergence.

References

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