Marcinkiewicz–Zygmund type inequalities with exponential weights

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Abstract

As is well known, Marcinkiewicz–Zygmund type inequalities are widely used in the study of the convergence of various approximation processes [6, 4, 1].

The classical inequalities were proved for trigonometric polynomials in 1937, whereas the algebraic case is more difficult and the first results were obtained by R. Askey in 1973 [2, 5]. In fact, the direct inequality

$$\left(\sum_{k=1}^{m} \Delta x_k \left|P_{lm}\left(x_k\right) u(x_k)\right|^p\right)^{1/p} \leq \mathcal{C}\left(\int_{-1}^{1} |P_{lm}(x)u(x)|^p \,\mathrm{d}x\right)^{1/p}, \quad 1 \leq p < \infty\,,$$

holds for any polynomial P_{lm} of degree lm (l fixed integer) with $\mathcal{C}$ depending only on p, $\Delta x_k = x_{k+1} - x_k$, x_k arbitrary arcsin distributed nodes and u a doubling weight [3], but the converse inequality requires more restrictive assumptions.

In this talk we discuss the case of non-doubling weights, namely exponential weights on bounded or unbounded intervals of the real line.

Keywords: Marcinkiewicz–Zygmund inequalities, orthogonal polynomials, exponential weights.

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