Krylov subspace based Fista-type methods for ill-posed problems

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Abstract

A group of iterative shrinkage-thresholding algorithms (ISTA) have been proposed for solving regularized linear inverse problems that arise in many applications in science and engineering. This group of methods, which can also be viewed as an extension of classical gradient algorithm, are simple and applicable to large-scale ill-posed problems. One of the difficulties that arise when applying those methods is the slow rate of convergence. Faster versions of ISTA such as fast iterative shrinkage-thresholding algorithms (FISTA), two-step ISTA (TWIST) and monotonic version of TWIST (MTWIST), are proposed to overcome this issue. While there are made improvements in terms of convergence, the computational cost of each iteration remains a concern, particularly for large scale problems. In this paper we propose a novel approach to speed up the convergence of the ISTA type methods, and decrease the computational cost by the aid of Krylov subspace methods. For applications such as image reconstruction, where the pixel values are nonnegative, one may impose nonnegativity constraint so that the reconstructed solution lies on the nonnegative orthant but such approach is known to cause slower convergence. We find a simply post nonnegative projection, i.e., setting all negative values of a solution to 0, improves the quality of the of the reconstructed solution. While working with regularization methods, the quality depends on the regularization parameter that balances the data fitting term and the regularization term. We propose a method that automatically selects the regularization parameter without significantly increasing the computational cost. This is especially useful for large-scale problems. Several numerical examples are presented to illustrate the efficiency of the proposed methods.

Keywords: Projected FISTA, Krylov subspace, nonnegativity constraint, regularization parameter choice rules, image reconstruction.