Internality of averaged Gauss quadrature rules for Jacobi measures modified by quadratic factor or divisor

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Abstract

Internal quadrature rules are essential when approximating integrals whose integrands are defined only on the support interval of the underlying measure. Earlier work established internality results for averaged Gauss and optimal averaged Gauss quadrature rules associated with Jacobi measures modified by a linear factor or divisor. In this paper, we extend those results to Jacobi measures altered by a quadratic factor or divisor of the form $(t-z)(t-\overline{z})$, where $z = x+iy$ and $y > 0$. We show that, under suitable assumptions and for sufficiently large numbers of nodes, the corresponding quadrature rules are internal. Particular attention is devoted to the cases of Chebyshev measures of the first and third kinds.

Keywords: Gaussian quadratures, Averaged Gauss quadrature, internality, Jacobi weights

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