Local Stability Analysis of Epidemic Models Using Gershgorin-type Localizations

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Abstract

Local stability analysis of epidemiological models typically relies on spectral properties of the Jacobian matrix evaluated at equilibrium points. In the recent literature, Gershgorin-type results have been successfully employed to derive sufficient conditions for stability without explicit eigenvalue computation. However, classical Gershgorin-based criteria often impose restrictive dominance conditions, which may be overly conservative for high-dimensional or strongly coupled systems.

We develop a systematic generalization of the Gershgorin approach by employing refined inclusion sets for eigenvalues, based on several Gershgorin-type theorems. These generalizations allow for a more accurate characterization of the spectrum of Jacobian matrices arising in compartmental epidemic models. As a consequence, we derive new sufficient conditions for local asymptotic stability of both disease-free and endemic equilibria that are strictly weaker than those obtained via classical Gershgorin discs. The proposed framework is applied to a class of multi-dimensional epidemiological models, demonstrating that stability can be guaranteed under less restrictive assumptions on model parameters.

In addition, besides the classical approaches, relying on induced norms such as the 1 and ∞ norms, which emphasize the maximal effect of individual components, we look back at the Frobenius norm, since modern high-dimensional applications increasingly require tools that capture the collective behavior of all matrix entries.

Keywords: Eigenvalue Localization, Geršgorin-type Theorems, Local Stability, Dynamical Systems, Epidemic Models