Remarks on r-parametric Hermite-Based Milne-Thomson Type Polynomials

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Abstract

In this study, we investigate r-parametric Hermite-based Milne-Thomson type polynomials and their generating functions. In [12], Kilar and Simsek defined the r-parametric Hermite type polynomials $\mathcal{K}(n; w, \overrightarrow{u}, r)$ as follows:

$$\mathcal{G}(t, w, \overrightarrow{u}, r) = \exp\left(wt + \sum_{j=1}^{r} u_j t^j\right) = \sum_{n=0}^{\infty} \mathcal{K}(n; w, \overrightarrow{u}, r) \frac{t^n}{n!}, \tag{1}$$

where $r \in \mathbb{N} = \{1, 2, 3, \ldots\}$, r-tuples $\overrightarrow{u} = (u_1, u_2, \ldots, u_r)$, $w = x + iy = (x, y)$, $u_1, u_2, \ldots, u_r$, $x, y \in \mathbb{R}$, the set of real numbers.

Further, the generating functions of r-parametric Hermite-based Milne-Thomson type polynomials $h(n, w, z; \overrightarrow{u}, r, a, b)$, $h_1(n, w, z; \overrightarrow{u}, r, a, b)$, and $h_2(n, w, z; \overrightarrow{u}, r, a, b)$ are given as follows:

$$\begin{aligned} M_1(t, w, z, \overrightarrow{u}, r, a, b) &= (b + f(t, a))^z \, \mathcal{G}(t, w, \overrightarrow{u}, r) \\ &= \sum_{n=0}^{\infty} h(n, w, z; \overrightarrow{u}, r, a, b) \frac{t^n}{n!}, \end{aligned} \tag{2}$$

$$\begin{aligned} M_2(t, w, z, \overrightarrow{u}, r, a, b) &= (b + f(t, a))^z \left(\mathcal{G}(t, w, \overrightarrow{u}, r) + \mathcal{G}(t, \overline{w}, \overrightarrow{u}, r)\right) \\ &= \sum_{n=0}^{\infty} h_1(n, w, z; \overrightarrow{u}, r, a, b) \frac{t^n}{n!}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} M_3(t, w, z, \overrightarrow{u}, r, a, b) &= (b + f(t, a))^z \left(\mathcal{G}(t, w, \overrightarrow{u}, r) - \mathcal{G}(t, \overline{w}, \overrightarrow{u}, r)\right) \\ &= \sum_{n=0}^{\infty} h_2(n, w, z; \overrightarrow{u}, r, a, b) \frac{t^n}{n!}, \end{aligned} \tag{4}$$

where $a, b \in \mathbb{R}$, $z \in \mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$, r-tuples $\overrightarrow{u} = (u_1, u_2, \ldots, u_r)$, $w = x + iy$ and $\overline{w} = x - iy$; the function $f(t, a)$ denotes analytic function or meromorphic function (*cf.* [12]).

By using the above generating functions, many computation identities and formulas have been obtained in [6]-[13]. In [12], modifications and unifications of the equations (3) and (4) were also investigated. We have also presented surface plots, graphical illustrations, and numerical values of these polynomials. These polynomials are closely related to many well-known special numbers and polynomials, such as the Hermite type polynomials, the generalized Hermite-Kampède Fèriet polynomials, the Milne-Thomson type polynomials, the Chebyshev polynomials, the Apostol-Bernoulli polynomials, the Apostol-Euler polynomials, the telephone numbers, the Bernstein polynomials, and parametric-type polynomials. Furthermore, many authors in the literature have studied these type polynomials; see [1]-[19].

Here, both some of their fundamental properties and some formulas related to them will be presented. In addition, some special cases of these relations are discussed in detail. These results provide important and useful contributions to both theoretical and computational studies. It is also shown that, in some special cases, these results reduce to many well-known results in the literature.

Keywords: r-parametric Hermite-based Milne-Thomson type polynomials, Generating functions, Special numbers and polynomials

References

1. Cesarano, C.: Hermite polynomials and some generalizations on the heat equations, Int. J. Syst. Appl. Eng. Dev. 8 (2014) 193–197.
2. Dattoli, G., Lorenzutta, S., Maino, G., Torre, A., Cesarano, C.: Generalized Hermite polynomials and supergaussian forms, J. Math. Anal. Appl. 203 (1996) 597–609.
3. Dattoli, G., Chiccoli, C., Lorenzutta, S., Maino, G., Torre, A.: Theory of generalized Hermite polynomials, Computers Math, Applic. 28 (1994) 71–83.
4. Dere, D., Simsek, Y.: Hermite base Bernoulli type polynomials on the umbral algebra, Russ. J. Math. Phys. 22 (2015), 1–5.
5. Hopper, A.T.: Generalized Hermite polynomials, Master's thesis, West Virginia University, 1961.
6. Kilar, N.: Generating functions of Hermite type Milne-Thomson polynomials and their applications in computational sciences, PhD thesis, University of Akdeniz, Antalya, 2021.
7. Kilar, N.: Integral formulas involving r-parametric Hermite-based Milne-Thomson type polynomials, in: Proceedings of the Western Balkan Conference on Mathematics and Applications, Tirana, 2024.
8. Kilar, N.: Relations and integration of Hermite-based Milne-Thomson and Fubini type polynomials, Akdeniz Univ. J. Sci. Eng. 1 (2024) 1–7.
9. Kilar, N.: Further results for Hermite-based Milne-Thomson type Fubini polynomials with trigonometric functions, Gazi Univ. J. Sci. Part A Eng. Innov. 11 (2024) 535–545.
10. Kilar, N.: On computational formulas for parametric type polynomials and its applications, J. BAUN Inst. Sci. Technol. 25 (2023) 13–30.
11. Kilar, N., Simsek, Y.: Identities and relations for Hermite-based Milne-Thomson polynomials associated with Fibonacci and Chebyshev polynomials, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. 115 (2021) 1–20.

12. Kilar, N., Simsek, Y.: Computational formulas and identities for new classes of Hermite-based Milne-Thomson type polynomials: analysis of generating functions with Euler's formula, Math. Methods Appl. Sci. 44 (2021) 6731–6762.
13. Kilar, N., Kim, D., Simsek, Y.: Formulae bringing to light from certain classes of numbers and polynomials, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. 117 (2023) 1–20.
14. Kim, T., Ryoo, C.S.: Some identities for Euler and Bernoulli polynomials and their zeros, Axioms 7 (2018) 1–19.
15. Kucukoglu, I.: Generating functions for multiparametric Hermite-based Peters-type Simsek numbers and polynomials in several variables, Montes Taurus J. Pure Appl. Math. 6 (2024) 217–238.
16. Leventeli, Y., Simsek, Y., Yilmazer, I.: Curve fitting for seismic waves of earthquake with Hermite polynomials, Publ. Inst. Math. (Beograd) (N.S.) 115 (2024) 101–116.
17. Ryoo, C.S.: Some identities involving Hermite Kampé de Fériet polynomials arising from differential equations and location of their zeros, Mathematics 7 (2019) 1–11.
18. Simsek, Y.: Formulas for Poisson–Charlier, Hermite, Milne-Thomson and other type polynomials by their generating functions and p-adic integral approach, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. 113 (2019) 931–948.
19. Srivastava, H.M., Masjed-Jamei, M., Beyki, M.R.: A parametric type of the Apostol–Bernoulli, Apostol–Euler and Apostol–Genocchi polynomials, Appl Math Inf Sci. 12 (2018) 907–916.