Orthogonality on the semicircle with respect to the complex-valued non-Hermitian inner product and applications

Marija P. Stanić
University of Kragujevac, Faculty of Science,
Department of Mathematics and Informatics
Radoja Domanovića 12, Kragujevac, Serbia
marija.stanic@pmf.kg.ac.rs

In this lecture we present a survey of the concept of orthogonality on the semicircle with respect to the complex-valued non-Hermitian inner product and the corresponding quadrature rules. Let D_+ be defined as $D_+ = \{z \in \mathbb{C} : |z| < 1, \mathrm{Im} z > 0\}$ and let Γ be a unit semicircle $\Gamma = \partial D_+ = \{z = \mathrm{e}^{\mathrm{i}\theta} : 0 \leq \theta \leq \pi\}$. Let $w(z)$ be a weight function which is positive and integrable on the open interval $(-1, 1)$, though possibly singularity at the endpoints, and which can be extended to a function $w(z)$ holomorphic in the half disc D_+. Orthogonal polynomials on the semicircle with respect to the complex-valued inner product

$$(f, g) = \int_\Gamma f(z)g(z)w(z)(\mathrm{i}z)^{-1}\,\mathrm{d}z = \int_0^\pi f(\mathrm{e}^{\mathrm{i}\theta})g(\mathrm{e}^{\mathrm{i}\theta})w(\mathrm{e}^{\mathrm{i}\theta})\,\mathrm{d}\theta$$

was introduced by Gautschi and Milovanović in [J. Approx. Theory 46 (1986), 230–250] (for $w(x) = 1$), where the certain basic properties were proved. We present applications of such orthogonality involving Gauss-Christoffel, anti-Gaussian and the generalized averaged Gaussian quadrature rules on the semicircle. The accuracy of such quadrature rules and applications are demonstrated through numerical examples.