On the Application of Kronrod Extensions of Gauss–Turan Quadrature

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Abstract

S. Li proposed a Kronrod-type extension to the Turán formula and showed that such an extension exists for any weight function. The existence and the uniqueness of Kronrod extensions of the corresponding Gauss-Turán quadrature formulas for the generalized Chebyshev weight function of the second kind were proven in the paper *Error estimations of Turán formulas with Gori-Micchelli and generalized Chebyshev weight functions (2018)*. The applications of the mentioned quadrature formulas in some areas of science and engineering are presented in this paper. It is shown that the methods developed by Spalević et al. provide stable error estimations and high precision, especially in the context of highly oscillatory integrands. Furthermore, their practical usage can be demonstrated in areas such as spectral analysis, statistical mechanics, mathematical physics, signal processing, quantum mechanics, theoretical optics, etc.

Keywords: Gauss-Turán quadrature formulas, Kronrod extension, Generalized Chebyshev weight function of the second kind, Error estimation.

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