

Error Estimation of Gaussian Quadrature Formulae for Jacobi Weight Functions and Some of Their Modifications

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Abstract

For analytic functions in a neighborhood of the real interval $[-1, 1]$, we study the remainder terms of Gauss quadrature rules with respect to Jacobi weight functions $\omega(x) = (1-x)^\alpha(1+x)^\beta$ and some of their modifications, namely

$$\omega_1(x) = \frac{(1-x)^\alpha(1+x)^\beta}{(v-x)}, \quad \omega_2(x) = (v-x)(1-x)^\alpha(1+x)^\beta,$$

where $\alpha, \beta > 1$, $v \in \mathbb{R}$, and $|v| > 1$. Quadrature formulas concerned with these kinds of weight functions have been considered recently by D. Lj. Djukić et al.

Keywords: Gaussian quadratures, polynomials, weight function, kernel, maximum, modulus

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