Gaussian quadrature rules with multiple nodes with respect to orthogonality on the semicircle

Marija P. Stanić[1], Tatjana V. Tomović Mladenović[1], and Aleksandra S. Milosavljević[1]

University of Kragujevac, Faculty of Science, Department of Mathematics and Informatics, Radoja Domanovića 12, Kragujevac, Serbia
{marija.stanic,tatjana.tomovic,aleksandra.milosavljevic}@pmf.kg.ac.rs

Abstract

Weighted orthogonality on the semicircle was introduced in 1987 in [1]. In that paper, orthogonality is considered with respect to a non-Hermitian inner product defined by

$$(f,g)=\int_{\Gamma} f(z)g(z)(\mathrm{i}z)^{-1}w(z)\mathrm{d}z=\int_{0}^{\pi} f(\mathrm{e}^{\mathrm{i}\theta})g(\mathrm{e}^{\mathrm{i}\theta})w(\mathrm{e}^{\mathrm{i}\theta})\mathrm{d}\theta,$$

where $\Gamma=\{z=\mathrm{e}^{\mathrm{i}\theta}\ :\ 0\leq\theta\leq\pi\}$. Gaussian quadrature rules associated with orthogonality on the semicircle were introduced in [2]. On the other hand, on the real line, Gaussian quadrature rules with multiple nodes have been studied in [3] and [4]. These are the so-called Gauss-Turán quadrature rules, defined with respect to s-orthogonality, and the Chakalov-Popoviciu rules, defined with respect to σ-orthogonality. We introduce Gaussian quadrature rules with respect to s-orthogonality and σ-orthogonality on the semicircle.

Keywords: Gaussian quadrature rules, Orthogonality on the semicircle, s-orthogonality, σ-orthogonality

References

1. Gautschi, W., Landau H.J., Milovanović, G.: Polynomials orthogonal on the semicircle, II, Constr. Approx. 3 (1987) 389–404.
2. Milovanović, G.: On polynomials orthogonal on the semicircle and applications, J. Comput. Appl. Math. 49 (1993) 193–199.
3. Gautschi, W., Milovanović, G.: S-orthogonality and construction of Gauss-Turán-type quadrature formulae, J. Comput. Appl. Math. 86 (1997) 205–218.
4. Milovanović, G.: Construction of Chakalov-Popoviciu's type quadrature formulae, Rendiconti del Circolo Matematico di Palermo 52 (1998) 625–636.