On Semantics for Preprobability Logic

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Abstract

Models for various systems of probability logic are based essentially on additivity condition. This condition is, in our opinion, incompatible with the elementary expectations and requirements of proof theory. In order to obtain a sound and complete system that is proof theoretically acceptable, and not far from the concept of probability, here we propose a weaker semantics than usual. Namely, instead of the condition of additivity, we start from the weaker condition of monotonicity, which promises the possibility of some normalization theorem for the system of natural deductions.

Roughly speaking, our model is defined as follows. Let For be the set of all propositional formulae and $\langle \mathbf{B}, ... \rangle$ is a finite Boolean algebra. Then a mapping $\pi : \text{For} \to \mathbf{B}$ will be an $\mathbf{NK}\pi$—*model* (or, simply, *model*), if it satisfies the following conditions:

(a) $\pi(\top) = 1$;

(b) if $A \to B$ in classical logic, then $\pi(A) \leq \pi(B)$ (monotonicity), and

(c) $\pi(\neg A) = \mathbf{c}(\pi(A))$, where $\mathbf{c}$ is the complementation operation in $\langle \mathbf{B}, ... \rangle$.

The system of preprobabilistic inference rules, denoted by $\mathbf{NK}\pi$, similarly as in Boričić [1], from Gentzen's **NK**, for natural deduction for classical logic (see Gentzen [2], Prawitz [3]), and π — for 'preprobability', consists of inference rules that allow the introduction, (I-), and elimination, (E-), of any logical connective. Each propositional formula A has its reprobabilistic counterpart A^r, for $r \in \mathbf{B}$, with intended meaning that 'preprobability of A is greater than or equal to r'.

The only axiom of $\mathbf{NK}\pi$ is A^1, for each classically provable proposition A, and, for instance, the inference rules dealing with conjunction look as follows:

$$\frac{A^r \quad B^s}{(A \wedge B)^{\min(r,s)}}(I\wedge) \qquad \frac{(A \wedge B)^r}{A^r}, \frac{(A \wedge B)^r}{B^r}(E\wedge)$$

Our semantical goal is to obtain a sound and complete system, and a normalizable system from a syntactic point of view.

Keywords: (Pre)probability logic, Syntax, Semantics, Proof theory. LaTeX

References

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