A global numerical method for approximating the solution of the fractional relaxation-oscillation equation

Maria Carmela De Bonis[1]

Department of Basic and Applied Sciences, University of Basilicata, Potenza, Italy
mariacarmela.debonis@unibas.it

Abstract

The relaxation-oscillation equation is the primary equation for describing the behaviour of physical phenomena that return to equilibrium after being disturbed. Fractional derivatives have been employed in order to represent slow relaxation and damped oscillation, obtaining the following fractional relaxation-oscillation equation

$$(D^\alpha y)(t) + \lambda y(t) = f(t), \quad t > 0, \tag{1}$$

where

$$(D^\alpha y)(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-x)^{m-\alpha-1} y^{(m)}(t) dt \tag{2}$$

is the Caputo's fractional derivative of order α with $m-1 < \alpha < m$ and $m = 1, 2$, $\lambda = \omega_0^2$ with ω_0 the natural frequency of the oscillator and f is an external force. For $0 < \alpha < 1$, (1) under the initial condition

$$y(0) = y_0$$

describes the relaxation with the power law attenuation; for $1 < \alpha < 2$, (1) under the initial conditions

$$y(0) = y_0, \quad y'(0) = y_1$$

represents the damped oscillation with viscoelastic intrinsic damping of oscillation [1, 2].

Several authors have studied the above equation and introduced numerical methods to approximate its solution (see, e.g [3, 4]).

We propose a Nyström method for the global approximation of its solution that is based on a suitable product quadrature rule obtained using Lagrange interpolation. Numerical tests showing the performances of the method will be presented, as well as comparisons with other existing numerical methods.

Keywords: Jacobi Polynomials, Riemann-Liouville fractional integrals, Caputo fractional Derivatives, Nyström method

References

1. Chen, W., Zhang, X.D., Korosak, D.: Investigation on fractional and fractal derivative relaxation-oscillation models, Int. J. Nonli. Sci. Numer. Simulat. 11 (1) (2010), 3–9.
2. Tofighi, A.: The intrinsic damping of the fractional oscillator, Phys. A 329 (2003), 29–34.
3. Wei, S., Chen, W.: A Matlab toolbox for fractional relaxation-oscillation equations, https://arxiv.org/abs/1302.3384 (2013)
4. Dimitrov, Y.: Approximations of the Fractional Integral and Numerical Solutions of Fractional Integral Equations, Commun. Appl. Math. Comput., 3 (2021), 545–569.