

Orthogonal Polynomials Related to a Complex-Valued Linear Moment Functional: Theory and Applications

Gradimir V. Milovanović¹

Serbian Academy of Sciences and Arts, 11000 Belgrade, Serbia
gvm@mi.sanu.ac.rs

Abstract

Motivated by certain applications in physics, in [4] the numerical calculation of the integral $\int_{-1}^1 f(x)W(x; \alpha, \beta)dx$, with complex parameters in the Jacobi weight function, under the assumption $\operatorname{Re}(\alpha) > -1$ and $\operatorname{Re}(\beta) > -1$, was considered. However, these applications in theoretical physics (calculation of the so-called transition amplitude [5, 6]) require further weakening of the conditions to the level of $\operatorname{Re}(\beta) = -1$. Let us mention that for complex parameters there are other types of orthogonal relations for Jacobi polynomials, such as the one in [3].

In order to obtain an adequate method for the aforementioned application, in this lecture we introduce orthogonal polynomials with respect to the linear complex-valued moment functional $L[z^k] = \mu_k$, $k = 0, 1, \dots$, defined in a suitable way (cf. [1], [2], [7]). Beside the existence and uniqueness we prove several properties, including zero distribution. In the second part we construct the corresponding quadrature formulas of Gaussian and anti-Gaussian type.

Keywords: Complex moment functional, orthogonal polynomial, moments, weight function, recurrence relation, zero distribution, Gaussian quadrature formula, nodes, Christoffel numbers

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