# Hardy-type inequalities and Green functions: interpolation and convexity approach

Dora Pokaz

University of Zagreb Faculty of Civil Engineering, Kačićeva 26, 10000 Zagreb, Croatia
dora.pokaz@grad.unizg.hr

## 1. Abstract

A function $f : [\alpha, \beta] \to \mathbf{R}$ is said to be $n$-convex if all its divided differences of order $n$ are non-negative, i. e. $[x_0, ..., x_n; f] \geq 0$ for all choices $x_i \in [\alpha, \beta]$. In particular, 0-convex functions are non-negative, 1-convex functions are non-decreasing, while 2-convex functions coincide with the classical notion of convexity.

In this work, we investigate convexity properties in the context of Hardy-type inequalities. The classical Hardy inequality is one of the fundamental results in the theory of inequalities and has motivated numerous generalizations in both pure and applied mathematics. It asserts that

$$\int_0^\infty \left( \frac{1}{x} \int_0^x f(t)\, dt \right)^p dx \leq \left( \frac{p}{p-1} \right)^p \int_0^\infty f^p(x)\, dx,\ p > 1 \tag{1}$$

for every non-negative function $f \in L^p(\mathrm{R}_+)$.

This inequality provides important estimates for weighted integrals and finds applications in functional analysis, operator theory, and differential equations. Owing to its flexibility, numerous variants of Hardy-type inequalities have been developed, depending on the underlying domain, weights, and measure structures.

We present results related to the Hardy-type inequalities formulated in a general setting on measure spaces $(\Sigma_i, \Omega_i, \mu_i)$, $i = 1, 2$, equipped with positive $\sigma$-finite measures and a measurable and non-negative kernel as introduced in [5].

**Keywords:** convex function, Hardy inequality, interpolating polynomial, Green function

## References

1. Agarwal, R.P., Wong, P.J.Y.: Error Inequalities in Polynomial Interpolation and their Applications, Kluwer Academic Publishers, Dordrecht, 1993.
2. P. Cerone, S.S. Dragomir, Some new Ostrowski-type bounds for the Cebysev functional and applications, J. Math. Inequal. 8(1) (2014), 159–170.

3. Gontscharoff, V.L.: Theory of Interpolation and Approximation of Functions, Gostekhizdat, Moscow, 1954.
4. Kaijser,S., Nikolova,L, Persson L.E., Wedestig, A.: Hardy-Type Inequalities via Convexity, Math. Inequal. Appl., 8 (2005), 403–417.
5. Krulić Himmelreich, K., Pečarić, J., Pokaz, D.: Inequalities of Hardy and Jensen, Element, Zagreb, 2013.
6. Krulić Himmelreich, K., Pečarić, J, Pokaz, D., Praljak, M.: Generalizations of Hardy-Type Inequalities by Montgomery Identity and New Green Functions, Axioms 2023, 12, 434, 1–13.
7. Krulić Himmelreich, K. Pečarić, J., Pokaz, D., Praljak M.: Hardy-type inequalities generalized via Montgomery identity, Montes Taurus J. Pure Appl. Math. 6 (3), 62–71, 2024.
8. Pokaz, D.: Inequality of Hardy-type for $n$-convex function via interpolation polynomial and Green functions, Math. Inequal.Appl., 26 (4) (2023), 965–976.
9. Whittaker, J.M.: Interpolation Function Theory, Cambridge Univ. Press, Cambridge, England, 1935.