Product integration rules for Chebyshev weight functions with Chebyshev abscissae

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Abstract

We review product integration rules, i.e., interpolatory quadrature formulae, relative to one of the Chebyshev weight functions of the first, second, third or fourth kind and based on the Chebyshev points, i.e., the zeros of the nth degree Chebyshev polynomial, of the first, second, third or fourth kind as well. Obviously, based on the weight function and the nodes chosen, there exist sixteen such rules, out of which one has to exclude the four well-known Gauss-Chebyshev quadrature formulae.

All the remaining twelve rules have weights given by explicit formulae and in all cases, except for two, the weights are all positive. Furthermore, we determine the precise degree of exactness and we compute the variance of the quadrature formulae, we examine their definiteness or nondefiniteness, and we obtain error bounds for these formulae either asymptotically optimal by Peano kernel methods or for analytic functions by Hilbert space techniques. In addition, the convergence of the quadrature formulae is shown not only for Riemann integrable functions on $[-1, 1]$, but also for functions having a monotonic singularity at one or both endpoints of $[-1, 1]$. Interestingly enough, five of these rules have the best possible degree of exactness for an interpolatory formula not of Gauss type (cf. [1], [2] and [3]).

Keywords: Product integration rules, Chebyshev weight functions, Chebyshev abscissae.

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