Sufficient Optimality Conditions and Duality for Lyapunov-type Optimization Problems with State Constraints

Jelena Vicanović[1], Ana Zekić[2], Ivana Sarić[3], and Jasmina Vujadinović[4]

[1] Faculty of Technology and Metallurgy , University of Belgrade, Karnegijeva 4
11000 Belgrade, Serbia
jvicanovic@tmf.bg.ac.rs
[2] Faculty of Technology and Metallurgy , University of Belgrade, Karnegijeva 4
11000 Belgrade, Serbia
azekovic@tmf.bg.ac.rs
[3] Faculty of Technology and Metallurgy , University of Belgrade, Karnegijeva 4
11000 Belgrade, Serbia
isaric@tmf.bg.ac.rs
[4] Military Academy, University of defence, Veljka Lukica Kurjaka 33
11042 Belgrade, Serbia
jasmina.fijuljanin@va.mod.gov.rs

Abstract

This paper investigates smooth Lyapunov-type continuous-time optimization problems, characterized by an integral objective functional subject to both isoperimetric (integral) and pointwise state constraints. The study is conducted within the framework of the $L_\infty([0,T],R^n)$ functional space, which is essential for ensuring the measurability and essential boundedness of candidate solutions.

The primary contribution of this work is three-fold: the establishment of sufficient optimality conditions based on generalized concavity (pseudoconcavity and quasiconcavity), the development of a formal duality theory (Mond-Weir and Wolfe models), and the extension of a computational framework for optimality verification. To guarantee optimality, we utilize the concepts of generalized concavity, proving that a candidate solution is optimal if the objective function is pseudoconcave and the constraints satisfy specific quasiconcavity assumptions.

Furthermore, we prove weak and strong duality theorems, utilizing corrected versions of the theorem of the alternative. Finally, the GLSymb framework, originally developed for computing candidate solutions through necessary optimality conditions [1], is extended to include the verification of sufficient conditions of optimality, providing a reliable symbolic-numerical procedure for validating global optima in Lyapunov-type problems.

Keywords: Countinuous-time programming, Lyapunov-type problems, Sufficient optimality conditions, Duality theory

References

1. Vicanović, J., Zekić, A: Necessary Optimality Conditions for Lyapunov-Type Optimization Problem with State Constraints., Mediterranean Journal of Mathematics 22.6 (2025) 170.