

Graph-Regularized Recovery of Student-Course Outcome Matrices via Structured Sparse Solvers

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Abstract

We consider the recovery of partially observed student-course outcome matrices arising from large academic records with repeated exam attempts and heterogeneous course interactions. To exploit the coupling across both students and courses, we formulate a graph-regularized matrix recovery model in which smoothness is imposed simultaneously over a student similarity graph and a course dependency graph. The optimality conditions lead to a large sparse linear system whose coefficient matrix splits as $A = D_W + A_0 + \gamma I$, where D_W is a diagonal mask, A_0 is a true Kronecker sum of two graph Laplacians, and $\gamma > 0$ is a Tikhonov parameter. We derive the exact spectrum of A_0 , sharp two-sided spectral bounds for A , and an explicit $O(nm(n+m))$ Kronecker-sum preconditioner P for which we prove the dimension-free condition-number bound $\kappa(P^{-1}A) \leq 1 + 1/\gamma$, independent of n , m , α , β , and the graph spectra. The proposed framework is evaluated on synthetic data, a real B.Sc. student-course matrix of shape $(560, 26)$ extracted from institutional records, and a period-level stacked dataset yielding a tensor of shape $(29, 2331, 239)$. On the real B.Sc. benchmark, the new Kronecker-sum preconditioner reduces the conjugate gradient iteration count from 63 (no preconditioner) and 44 (Jacobi) to 13, a $4.8\times$ speedup over no preconditioning and a $3.4\times$ speedup over Jacobi, while preserving the holdout error up to numerical precision. The measured iteration count is consistent with the predicted bound $\sqrt{\kappa(P^{-1}A)} \leq \sqrt{6} \approx 2.45$.

Keywords: graph-regularized recovery, Kronecker-sum systems, Sylvester equations, preconditioned conjugate gradients, sparse matrix completion, numerical linear algebra

References

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