Remarks on Bernstein base functions with their applications

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Abstract

This study will critique recent research on Bernstein basis functions and the construction of their generating functions, and will offer some predictions about their future applications. Because, as we can observe in the literature, Bernstein basis functions provide resources that can be applied to almost all other branches of science, including most branches of mathematics. Perhaps because Bernstein basis functions serve as models in B-spline theory, which includes Bezier curves, as well as in affine combination theory and other fields, these basis functions constitute a fundamental focus of many researchers' work.

There are various different families of Bernstein-type basis functions. These basis functions are related to the Schurer polynomials, the Kantorovich polynomials, the Stancu polynomials, the q-Bernstein polynomials, the Durrmeyer polynomials, the Favard-Szasz-Mirakjan operators, the Baskakov operators, and the Balazs-Szabados operators, etc (*cf.* [1]-[17]).

The Bernstein bases functions are defined by

$$B_b^n(w) = \frac{n!}{b!(n-b)!} w^b (1-w)^{n-b},$$

where n is nonnegative integer with $0 \leq b \leq n$; generating functions for the function $B_b^n(w)$ are given by

$$\frac{(tw)^b}{b!} e^{-t(w-1)} = \sum_{n=0}^{\infty} \frac{B_b^n(w)}{n!} t^n \tag{1}$$

(*cf.* [1], [17]-[19], [8], [12]). Note that there is one generating function for each value of w and b.

The generalized falling factorial $w^{(m,\theta)}$, which is known as homogeneous polynomial in w and θ of degree m, with increment θ is defined by

$$w^{(m,\theta)} = \prod_{v=0}^{m-1} (w - v\theta) \tag{2}$$

for m is a positive integer, with the convention $w^{(0,\theta)} = 1$ (see [15, p. xxii]). It is clear to see that

$$w^{(m,0)} = w^m$$

and

$$(w)^{(m)} := (w)^{(m,1)},$$

(cf. [16]).

Kim and Kim [9] defined the following generating function for the degenerate Bernstein polynomials

$$\frac{(w)^{(k,\theta)}}{k!} t^k (1+\theta t)^{\frac{1-w}{\theta}} = \sum_{n=0}^{\infty} \frac{B_{k,n}(w;\theta)}{n!} t^n.$$

Here we note that

$$\frac{t^k}{k!} \lim_{\theta \to 0} (w)^{(k,\theta)} (1+\theta t)^{\frac{1-w}{\theta}} = \frac{(tw)^k}{k!} e^{t(1-w)}.$$

Results and Contributions:

We give many new results for degenerate Bernstein type polynomials via their generating functions. Our results are related to the the degenerate Bernoulli polynomials, the Bernoulli polynomials of higher order, the Stirling numbers etc.

Our results have the potential to serve as a resource not only in the theory of Bernstein polynomials and B-spline, but also in other scientific disciplines.

Keywords: Bernstein basis functions, Bernoulli Polynomials of higher order, Generating functions, Stirling numbers.

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