Gauss-type quadratures with quasi-prescribed nodes

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Abstract

Consider the numerical approximation of integrals over a finite interval $[a, b]$ by Gauss-type quadrature rules with prescribed nodes $x_k \in R \setminus (a, b)$, $k = 1, 2, ..., m$. In particular, if these nodes coincide with the zeros of the integrand, one can effectively obtain an n-point quadrature formula that achieves a degree of exactness $2n - 1 + m$.

Motivated by this observation, we propose a decomposition of the given integral into two parts. The first part can be evaluated analytically and therefore does not contribute to the quadrature error. The second part has x_k, $k = 1, 2, ..., m$, as zeros of a suitably modified integrand. This approach enables the construction of Gauss-type quadrature rules with enhanced accuracy for the remaining integral. In the proposed framework, the nodes x_k, $k = 1, 2, ..., m$, are referred to as quasi-prescribed, since they do not appear explicitly in the final quadrature formula.

Particular attention is given to cases corresponding to Gauss-Radau and Gauss-Lobatto formulas. Several numerical examples are presented to illustrate the theoretical results.

Keywords: Gauss-type quadratures, (quasi) prescribed nodes, Radau-type rules, Lobatto-type rules

References

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